Guaranteed Fault Containment and Local Stabilization in Routing

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***Abstract*— We formulate concepts that characterize network properties in the presence of high-frequency faults, and we present CPV, a path-vector routing protocol that locally contains high-frequency faults and locally stabilizes. Local containment enables CPV to guarantee that distant nodes be unaffected by faults. Local stabilization enables CPV to stabilize the network within time depending only on the perturbation size instead of the network size. In CPV, the distance to which the state of a node propagates is proportional to the time the state remains valid. These properties are achieved by reacting to a new fault only after first containing the response to the previous fault. In addition to analytically proving these properties, we evaluate CPV by simulating Internet-type networks with up to 75 autonomous systems; we observe that CPV reduces the number of fault-affected nodes by a factor of 71 and the network convergence time by a factor of 9.2 when compared with BGP.**

***Keywords*— unanticipated faults, high-frequency faults, fault containment, local stabilization, path-vector routing, Internet**

1 Introduction

To guarantee high stability and availability of large scale networks and distributed systems, an ideal is to withstand failure or compromise of one or more regions in a network without impacting a large part of the network [7, 26, 3]. To this end, formal models have been proposed to analyze the properties of networked and distributed systems in terms of locally containing the impact of faults [8, 4, 3]. These models have largely focused on cases where faults stop occurring after certain moment in time, or faults occur at low frequencies. In practice, however, faults may keep occurring at *high frequencies*, and the interval between faults may be shorter than the time taken for the network to stabilize [26]. Therefore, we need to formulate new models that can serve as the foundation for analyzing system properties in the presence of high-frequency faults.

To improve system stability in the presence of faults, existing networked systems (such as the Internet) have deployed mechanisms for containing the impact of faults [23, 24]. Depending on how their parameters are configured, these mechanisms usually can only deal with faults of specific patterns[1]. Nevertheless, complex interactions between different network components may generate *unanticipated* faults, especially when networks work in stressful conditions. Consequently, most existing mechanisms do not guarantee fault containment in the presence of these unanticipated faults (which may be of high-frequency too).

To given an example, let us consider inter-domain routing in the Internet via path-vector protocol BGP [26]. Under the unanticipated Code Red/Nimda worm attack, memory overflow and BGP session reset caused some routers to repeatedly fail-stop and rejoin at frequencies as high as once every minute [26]. Even though instability-suppression timers (controlled by parameters such as *MinRouteAdvertisementInterval* and *MinASOriginationInterval* [23]) and route-flap-damping are used in BGP, they can only contain faults of specific patterns, and they do not guarantee fault containment in all cases. For instance, a corrupted route announced more than *MinRouteAdvertisementInterval* later than the previous announcement can propagate from a node to others, and oscillating routes can propagate from a node before the associated penalty value exceeds the cut-off threshold in route-flap-damping. Therefore, faults at some edge routers propagate across the whole Internet in the presence of the worm attack [26], thus significantly decreasing the stability and availability of the network.

To provide dependable services, therefore, it is desirable that networks always contain the impact of high-frequency faults, whether or not anticipated, locally around where they occur; we refer to this property as *continuous containment*. It is also desirable that, once faults do not occur any more[2], networks converge quickly and within time as a function of the degree of fault perturbation instead of the network size; we refer to this property as *local stabilization*. Toward these objectives, we need to formulate models that characterize system behaviors and to design mechanisms that guarantee continuous containment and local stabilization in the presence of high-frequency unanticipated faults.

Related work. The concepts of fault containment and local stabilization have been discussed in [8], [3], and [4]. These concepts are proposed only for the cases where faults either stop or only state corruption can keep occurring. Therefore, they do not apply to the cases where

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[1]Fault patterns are defined by properties such as temporal distribution (e.g., interval between two consecutive faults) and spatial distribution (e.g., graph distances among nodes where faults occur) of faults.

[2]Or, no fault occurs until after a period significantly longer than the time taken for the network to stabilize from the previous faults.

complex unanticipated faults keep occurring at high frequencies.

To locally contain faults in distance-vector routing, protocol LSRP [3] has been proposed. Having to avoid forming routing loops, however, LSRP only guarantees fault containment and local stabilization for scenarios where faults happen at low frequencies and networks get enough time to stabilize from one fault before another one occurs. Therefore, LSRP does not guarantee fault containment when faults keep occurring at high frequencies.

In [8], algorithms are proposed to contain a single state corruption during the stabilization of a spanning tree; these algorithms do not focus on and thus do not deal with high-frequency faults, node fail-stop, and node join. In [4, 11], protocols are proposed to contain observable variables (i.e., variables that are read by at least another component) in the presence of state corruptions, for the problem of *broadcast* and *persistent bit* respectively; these protocols do not focus on containing internal variables (i.e., variables that are neither read nor written by any other component), thus corrupted internal variables can propagate unboundedly. In this paper, we focus on containing both observable and internal variables, and we consider arbitrary high-frequency faults (as a result, low frequency faults can also be easily dealt with).

For containing observable variables [11], self-stabilization in unidirectional networks [2], and fault containment in distance-vector routing [3], the technique of letting different control messages propagating at different speed has been used. This technique will also be applied in this paper. As we will see when we develop our protocol, however, whether to contain internal variables and high-frequency faults determines the number of message types required (i.e., 3 instead of 2) and how the propagation of different messages coordinates with one another. Of course, differences in problem definitions also impose different requirements on protocol design; for instance, loop-freedom is required in this paper but not in [11] and [2].

To improve the stability of BGP, instability-suppression timers and route-flap-damping are used [23, 24]. Nevertheless, they assume certain inter-fault intervals and fault predictability. Therefore, they deal with faults of certain patterns, but they do not guarantee the containment of unanticipated high-frequency faults, as experienced in Internet [26] and observed in [3]. (We also re-produce the phenomenon via simulation in Section 7.)

To improve BGP convergence speed after fault occurrence, various mechanisms have been proposed [22, 15, 5, 28, 20]. These mechanisms do not focus on fault containment, but to further improve network stability and availability, they can be applied together with the mechanism to be developed in this paper.[3]

Contributions of the paper. To build the foundation for studying and to precisely characterize system properties in the presence of unanticipated high-frequency faults, we formulate the notions of perturbed node, contaminated node, perturbation size, contamination range, $\mathcal{F}$-containment, and $\mathcal{F}$-stabilization (see Section 4). These concepts are generally applicable to networking and distributed computing problems.

For the problem of path-vector routing, we design CPV, a path-vector routing protocol that contains high-frequency faults, whether or not anticipated, and is locally stabilizing. In the presence of faults, the properties of CPV are as follows:

- The only nodes that are affected by a fault are those within $O(p)$ distance from the fault-perturbed region, where p is the size of the perturbed region.
- The distance to which the state of a node propagates is proportional to the sojourn time of the state (i.e., the time for which the state remains valid), and as a result, the more unstable a node is, the shorter is the distance to which its state propagates.

Once faults stop occurring (either indefinitely or for a long enough period), the network converges to a legitimate state within $O(\Gamma(p'))$ time, where p' is the perturbation size of the faults and Γ is a function dependent on the routing policy used in the network[4].

CPV achieves these properties via the following design pattern. When a new fault occurs, before generating a stabilization wave to correct the network routes, CPV first contains, by engaging a containment wave, the effect of the stabilization wave resulting from the previous fault. The containment wave propagates faster than the stabilization wave to this end. To deal with the case where the containment wave is itself propagated in error (for instance, if yet another fault happens before the stabilization wave corresponding to the new fault is generated), CPV generates an undo-containment wave that propagates even faster than the containment wave. This wave is self-correcting. We find that this pattern is generally applicable to other networking and distributed computing problems where diffusing computation is used.

We analytically evaluate the properties of CPV using the concepts defined in the paper. We also evaluate CPV by simulating Internet-type networks with up to 75 autonomous systems (ASes). The simulation corroborates our analysis by showing that, in the presence of high-frequency faults, CPV reduces the number of fault-affected nodes by a factor of 71, the network convergence time by a factor of 9.2, and the number of control messages by three orders of magnitude when compared with BGP.

Organization of the paper. In Section 2, we present the network model, the protocol notation, and the fault model. We also briefly recall BGP. We give an example of fault propagation in Section 3. Then, we formulate, in Section 4, the concepts of fault containment and local stabilization in the presence of high-frequency faults. We discuss the design of CPV in Section 5. We analyze the properties of CPV in Section 6 and present the simula-

[3] We discuss this in more detail in Section 8.

[4] Γ is linear when the shortest-path-first route ranking policy is used.

tion results in Section 7. We discuss issues related to the application of CPV in Section 8, and we make concluding remarks in Section 9. Finally, we present in the Appendix the proofs of the theorems in this paper.

2 Preliminaries

In this section, we present the network model, the protocol notation, and the fault model. We also briefly recall BGP.

Network model. A network G is an undirected graph (V, E, P), where V is the set of nodes (i.e., BGP speakers), E is the set of links, and P is the function that defines the routing policies (e.g., route ranking policy as discussed at the end of this section) of each node. In this paper, we only consider routing policy functions by which BGP converges [10]. V is divided into several subsets, each of which is an autonomous system (simply denoted as AS hereafter). Each node has a unique node-id, and all the nodes in the same AS have the same AS-id. For a node i, the id of its AS is denoted by $i.as$. For any two nodes i and j, (i, j) is in E if i and j can communicate with each other directly or via some external mechanism (such as intra-AS/intra-domain routing in the Internet).

Message transmission between nodes is reliable, and message passing delay across a link e is bounded from above and from below by $e.U$ and $e.L$ respectively. There is a clock at each node; the ratio of clock rates between a node i and any of its neighbors is bounded from above by $i.\alpha$, but no extra constraint on the absolute values of clocks is enforced. (Note: $e.U > 0$, $i.\alpha \geq 1$, $e.U$ tends to be quite small given today's high speed networks, and $i.\alpha$ tends to be quite close to 1 given today's high precision clocks.)

For clarity of presentation, we only consider one destination d, an address prefix representing a set of nodes in an AS $d.as$. (Our protocol readily applies when there are multiple destinations.)

Protocol notation. We write protocols using a variant of the Abstract Protocol notation [9]. At each node, the protocol consists of a finite set of variables and actions. Each action consists of three parts: guard, guard hold-time, and statement. For convenience, we associate a unique name with each action. Thus, an action has the following form:

$$\langle name \rangle :: \langle guard \rangle \xrightarrow{h} \langle statement \rangle$$

The guard is either a boolean expression over the protocol variables of the node or a message reception operation; h is the guard hold-time ($h \geq 0$); the statement updates zero or more protocol variables of the node and/or sends out some message(s). If $h = 0$, we write the action in the following form:

$$\langle name \rangle :: \langle guard \rangle \longrightarrow \langle statement \rangle$$

For an action whose guard is a message reception operation, its guard hold-time must be 0.

For an action named a, its guard hold-time is denoted by $h.a$. An action a is enabled at time t if the guard of a evaluates to true at t. An action a is executed at time t only if a is continuously enabled from time $(t - h.a)$ to t. To execute an action, the corresponding statement is executed atomically.

Fault model. A node or a link is *up* if it functions correctly, and it is *down* if it fail-stops. We consider the following network faults: an up node or link can fail-stop and become down; a down node or link can become up and join the network; and the routing policies of a node can change. The interval between any two faults can be any non-negative value.

Border Gateway Protocol. BGP is a path-vector routing protocol (where a node maintains the complete AS-level path to each destination) used to coordinate routing among ASes in the Internet [23]. In BGP, UPDATE messages are passed between nodes to convey routing information. BGP UPDATEs are route records that include the following attributes (among others):

nlri	:	network layer reachability information (i.e., the destination address);
next_hop	:	the next hop;
AS_path	:	ordered list of ASes traversed, with more-recently-visited ASes in front of less-recently-visited ASes;
local_pref	:	local preference;
med	:	multi-exit discriminator.

Each route r is associated with a 3-tuple $rank(r)$, defined as $\langle r.local\_pref, \frac{1}{|r.as\_path|}, \frac{1}{r.next\_hop} \rangle$. For the destination d, a node i chooses its route via the following two steps [23]:

- First, among all the routes learned from a neighboring AS, i only considers the route with the lowest *med* value;
- Second, for all the routes to be considered, i ranks them in lexical order by $rank(\cdot)$, and i selects as its route the one with the highest rank.

Given a route r available to a node i, attribute $r.local\_pref$ is determined by the *route ranking policy* of i. For convenience, we call the ranking policy that assigns $r.local\_pref$ to a constant value the *shortest-path-first policy* or the *SPF policy*, where a route with the shortest AS-path ranks the highest.

Besides route ranking policy, BGP uses import and export policies. The *import policy* of a node i defines the set of import neighbors of i whose routes are accepted by i; the *export policy* of i defines the set of export neighbors of i to which i announces its route.

3 Fault propagation in BGP

To see how faults propagate unboundedly in BGP, we consider the simple network shown in Figure 1, where d is the destination to which the other nodes maintain a route. Suppose d fail-stops at a state where all other nodes have learned their routes to d. Then e will withdraw its route to d and send a route-withdrawal message to f. Now suppose

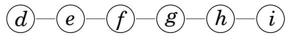


Figure 1: A simple line network. For simplicity, each node in the figure also represents its AS.

d rejoins the network before f withdraws its route. Then nodes f, g, h, and i should, ideally, not withdraw their routes anymore. Nevertheless, the fault history and the interval between d fail-stopping and d rejoining may be such that the instability-suppression timers as well as the route-flap-damping in BGP do not prevent nodes f, g, h, and i from withdrawing their routes (even though these nodes will learn later the same routes again). Thus, in this fault scenario, the faults propagate to all the nodes in the network, whereas the faults should ideally only affect e. Due to the fault propagation, the time taken for the network to stabilize after d rejoins is a function of the network size instead of the minimum number of nodes that should have been affected (which is 1 in this case).

To simplify presentation, we used a simple network and a simple fault scenario in the above discussion. In practice, networks are more connected and multiple paths exist for a destination, which can incur more network instability [28]; there are also more complex fault scenarios, with varying frequencies of fault occurrence and varieties of faults (including state corruption, misconfiguration, software bugs, etc.) [14, 16]. Therefore, the negative impact of fault propagation is even more severe in practice (as shown in Section 7 and in [26]). Thus, unbounded fault propagation in networks should be avoided.

4 Continuous containment and local stabilization

Toward establishing the foundation for studying and toward precisely characterizing system properties in the presence of high-frequency faults, we formulate in this section the notions related to continuous containment and local stabilization (e.g., perturbed node, contaminated node, perturbation size, contamination range, $\mathcal{F}$-containment, and $\mathcal{F}$-stabilization).

We regard a *system* as the union of a network and the protocols running on it. In the presence of faults, a network G may change in the sense that its topology or routing policy function changes, where the topology of G is the subgraph $G'(V', E')$ of $G(V, E)$ such that $V' = \{i : i \in V \wedge i \text{ is up}\}$ and $E' = \{(i,j) : i \in V' \wedge j \in V' \wedge (i,j) \in E \wedge (i,j) \text{ is up}\}$. To reflect changes in network topology and routing policy function, we regard the state of a system as the union of the network topology, the routing policy function, and the state of all the up nodes, with the state of a node being the values of the variables maintained at the node. At a system state q, the network topology is denoted by $G.q(V.q, E.q)$, and the state of a node j is denoted as $j.q$.

We characterize the behavior of a system in the presence of faults by the *system history*. A *system history* $\mathcal{H}$ is either a finite sequence q_0, (e_1, t_1), q_1, (e_2, t_2), $\ldots$, q_n, or an infinite sequence q_0, (e_1, t_1), q_1, (e_2, t_2), $\ldots$, q_{k-1}, (e_k, t_k), q_k, $\ldots$, of alternating system states (i.e. $q_0, q_1, \ldots$) and events (i.e. $e_1, e_2, \ldots$), where

- An event is either the execution of a protocol action or the occurrence of a fault;
- For every $k \geq 1$, $t_k \leq t_{k+1}$, and each state transition $q_{k-1}, (e_k, t_k), q_k$ means that e_k at time t_k changes the system state from q_{k-1} to q_k; and
- For any two pairs (e_k, t_k) and $(e_{k'}, t_{k'})$ in $\mathcal{H}$ ($k \neq k'$), if e_k and $e_{k'}$ occur at the same node, then $t_k \neq t_{k'}$ (i.e., at most one event can occur at a node at any time).

$\mathcal{H}$ is a finite sequence only if it ends with a state q_n such that there is no action enabled at q_n and no fault occurs after the system reaches q_n.

A subsequence α of a system history $\mathcal{H}$ is called a *history segment* if α starts and ends with a state. A history segment β is called a *history prefix* if β starts with the initial state q_0. For convenience, we denote as $\mathcal{H}(q)$ a history prefix ending with a state q. A history segment γ starting at a state q_k is called a *computation starting at* q_k if γ is the suffix of $\mathcal{H}$ starting at q_k and every event in γ is the execution of a protocol action (i.e., no fault occurs in γ).

Given a network, a protocol specification defines a set of legitimate states. For example, given a network topology, a routing policy function, and a destination, the specification of CPV determines a set of legitimate states, each of which specifies the route of every node. For a self-stabilizing protocol, each computation C_k starting at an arbitrary state q_k determined a converged state $L(q_k, C_k)$. Whenever a fault changes a system state from q_k to q_{k+1}, the instance of computation changes in the sense that the computation C_{k+1} starting at q_{k+1} is different from C_k. Then, given a state q_k with a history prefix $\mathcal{H}(q_k)$, we regard as a *protocol execution* $\mathcal{E}(q_k)$ a set of computations each of which specifies a computation $C(q_{k'}, \mathcal{E}(q_k))$ for a different state $q_{k'}$ in $\mathcal{H}(q_k)$ that is either the initial state or a state reached immediately after a fault occurs. Then, given an arbitrary state q_k and an arbitrary protocol execution $\mathcal{E}(q_k)$, we define the *stabilization-set of* q_k *under* $\mathcal{E}(q_k)$, denoted by $S(q_k, \mathcal{E}(q_k))$, as the set of nodes that need to change state in order for the network to stabilize from q_k, i.e., $S(q_k, \mathcal{E}(q_k)) = \{j : j \in V.q_k \wedge j.q_k \neq j.L(q_k, C(q_k, \mathcal{E}(q_k)))\}$. (Of course, if q_k is a legitimate state, $S(q_k, \mathcal{E}(q_k)) = \emptyset$.)

If an event e occurring at time t_k changes the system state from q_{k-1} to q_k under a protocol execution $\mathcal{E}(q_k)$, we define the *corruption set of* e *at* t_k, denoted by $cpt(e, t_k, \mathcal{E}(q_k))$, as the set of nodes that need not change state in order for the network to stabilize from q_{k-1}, but need to change state in order for the network to stabilize from q_k, i.e., $cpt(e, t_k, \mathcal{E}(q_k)) = S(q_k, \mathcal{E}(q_k)) \setminus S(q_{k-1}, \mathcal{E}(q_k))$. In addition, if e is not a state corruption, we define the *correction set of* e *at* t_k, denoted by $cct(e, t_k, \mathcal{E}(q_k))$, as the set of nodes in $V.q_k$ that need to change state in order for the network to stabilize from q_{k-1}, but need not change state in order for the network to stabilize from q_k, i.e., $cct(e, t_k, \mathcal{E}(q_k)) = (S(q_{k-1}, \mathcal{E}(q_k)) \setminus S(q_k, \mathcal{E}(q_k))) \cap V.q_k$. If e is a state corruption, we define $cct(e, t_k, \mathcal{E}(q_k))$ as $\emptyset$.

Perturbation vs. contamination. For every node $j \in cpt(e, t_k, \mathcal{E}(q_k))$, we regard j as *perturbed* by e if e is a fault, and we regard j as *contaminated* via e if e is the execution of a protocol action; for every node $j \in cct(e, t_k, \mathcal{E}(q_k))$, we regard j as *corrected* by e. For instance, in the example discussed in Section 3, when d fail-stops, $e, \ldots, i$ are perturbed by the fail-stop of d; when d rejoins before f withdraws its route, $f, \ldots, i$ are corrected by the join of d; when BGP is used, $f, \ldots, i$ continue to withdraw their routes after d rejoins, thus $f, \ldots, i$ are contaminated via BGP actions.

Given a system state q_k with a history prefix $\mathcal{H}(q_k)$ and a protocol execution $\mathcal{E}(q_k)$, a node i is *perturbed at* q_k if either of the following conditions hold:

- q_k is the initial state (i.e., $k = 0$), and i needs to change state in order for the network to stabilize.
- i has been perturbed by a fault at some point, and neither i has been corrected by a fault nor has the network reached a legitimate state ever since.

A node i is *contaminated at* q_k if i has been contaminated at some point and has not been corrected ever since. Formally,

Definition 1 (Perturbed node) *Given a protocol execution $\mathcal{E}(q_k)$, and a network state q_k with history prefix $\mathcal{H}(q_k)$ $(k \geq 0)$, a node i $(i \in V.q_k)$ is perturbed at q_k if and only if*

- $i \in S(q_k, \mathcal{E}(q_k))$, *when* $k = 0$
- $PT(i, q_0, q_k, \mathcal{H}(q_k), \mathcal{E}(q_k)) = true$, *when* $k > 0 \wedge \neg(\exists k' : q_{k'} \in \mathcal{H}(q_k) \wedge q_{k'} \in Q_L(q_{k'}))$
- $PT(i, LL(q_k, \mathcal{H}(q_k)), q_k, \mathcal{H}(q_k), \mathcal{E}(q_k)) = true$, *when* $k > 0 \wedge (\exists k' : q_{k'} \in \mathcal{H}(q_k) \wedge q_{k'} \in Q_L(q_{k'}))$

where

$$\begin{aligned}
&PT(i, q_m, q_n, \mathcal{H}(q_n), \mathcal{E}(q_k)) \equiv \\
&\quad q_m \in \mathcal{H}(q_n) \wedge \\
&\quad ((e_n \textit{ is a fault} \wedge i \in cpt(e_n, t_n, \mathcal{E}(q_k))) \vee \\
&\quad\ (\exists k' : m \leq k' < n \wedge i \textit{ is perturbed at } q_{k'} \wedge \\
&\qquad\quad \neg(\exists j' : k' < j' \leq n \wedge (e_{j'}, t_{j'}) \in \mathcal{H}(q_n) \wedge \\
&\qquad\qquad e_{j'} \textit{ is a fault} \wedge i \in cct(e_{j'}, t_{j'})))) \\
&LL(q_k, \mathcal{H}(q_k)) = q_j \textit{ such that} \\
&\quad q_j \in \mathcal{H}(q_k) \wedge q_j \in Q_L(q_j) \wedge \\
&\quad\ \neg(\exists j' : j < j' \leq k \wedge q_{j'} \in \mathcal{H}(q_k) \wedge q_{j'} \in Q_L(q_{j'})) \\
&Q_L(q_j) = \textit{the set of legitimate states corresponding} \\
&\quad \textit{to the network topology and routing policy} \\
&\quad \textit{function at } q_j.
\end{aligned}$$

Definition 2 (Contaminated node) *Given a protocol execution $\mathcal{E}(q_k)$, and a network state q_k with history prefix $\mathcal{H}(q_k)$ $(k \geq 0)$, a node i $(i \in V.q_k)$ is contaminated at q_k if and only if*

- $CT(i, q_0, q_k, \mathcal{H}(q_k), \mathcal{E}(q_k)) = true$, *when* $k > 0 \wedge \neg(\exists k' : q_{k'} \in \mathcal{H}(q_k) \wedge q_{k'} \in Q_L(q_{k'}))$
- $CT(i, LL(q_k, \mathcal{H}(q_k)), q_k, \mathcal{H}(q_k), \mathcal{E}(q_k)) = true$, *when* $k > 0 \wedge (\exists k' : q_{k'} \in \mathcal{H}(q_k) \wedge q_{k'} \in Q_L(q_{k'}))$

where

$$\begin{aligned}
&CT(i, q_m, q_n, \mathcal{H}(q_n), \mathcal{E}(q_k)) \equiv \\
&\quad q_m \in \mathcal{H}(q_n) \wedge \\
&\quad ((e_n \textit{ is not a fault} \wedge i \in cpt(e_n, t_n, \mathcal{E}(q_k))) \vee \\
&\quad\ (\exists k' : m \leq k' < n \wedge i \textit{ is contaminated at } q_{k'} \wedge \\
&\qquad\quad \neg(\exists j' : k' < j' \leq n \wedge (e_{j'}, t_{j'}) \in \mathcal{H}(q_n) \wedge \\
&\qquad\qquad i \in cct(e_{j'}, t_{j'})))) \\
&LL(q_k, \mathcal{H}(q_k)) = q_j \textit{ such that} \\
&\quad q_j \in \mathcal{H}(q_k) \wedge q_j \in Q_L(q_j) \wedge \\
&\quad\ \neg(\exists j' : j < j' \leq k \wedge q_{j'} \in \mathcal{H}(q_k) \wedge q_{j'} \in Q_L(q_{j'})) \\
&Q_L(q_j) = \textit{the set of legitimate states corresponding} \\
&\quad \textit{to the network topology and routing policy} \\
&\quad \textit{function at } q_j.
\end{aligned}$$

Intuitively, these definitions imply that, a perturbed node remains perturbed until it is corrected by a fault or the network reaches a legitimate state; a contaminated node remains contaminated until it is corrected by either a fault or the execution of a protocol action. For instance, in the example discussed in Section 3, at the state immediately after d fail-stops, $e, \ldots, i$ are all perturbed; at the state immediately after d rejoins, only e is perturbed, since $f, \ldots, i$ are corrected by the rejoining of d; at the state where f withdraws its route, f is contaminated.

The number of perturbed nodes at a system state, which we define as the *perturbation size*, reflects the severity of the impact that faults have on the system. Formally,

Definition 3 (Perturbation size) *Given a system state q_k with a history prefix $\mathcal{H}(q_k)$ and a protocol execution $\mathcal{E}(q_k)$, the perturbation size at q_k, denoted by $\mathcal{P}(q_k, \mathcal{H}(q_k), \mathcal{E}(q_k))$, is the number of nodes that are perturbed at q_k. That is, $\mathcal{P}(q_k, \mathcal{H}(q_k), \mathcal{E}(q_k)) = |\{i : i \in V.q_k \wedge i \textit{ is perturbed at } q_k\}|$.*

A set S of nodes are *contiguous* at a system state q if $S \subseteq V.q$ and the subgraph of $G.q(V.q, E.q)$ on S is connected, i.e., the graph $G'(V', E')$ is connected, where $V' = S$ and $E' = \{(i,j) : i \in S \wedge j \in S \wedge (i,j) \in E.q\}$. We regard a maximal set of perturbed nodes that are contiguous as a *perturbed region*. We also regard a contaminated node i as being *contaminated by a perturbed region*, via the execution of a protocol action a, if a is executed at i because of the state changes at a node j that is either in or contaminated by the perturbed region. Then, given a perturbed region S_p at a state q_k, the maximum distance from the nodes contaminated by S_p to S_p, which we define as the *contamination range of S_p at q_k*, reflects the severity of fault propagation from S_p at q_k. Formally,

Definition 4 (Contamination range) *Given a perturbed region S_p at a state q_k, the contamination range of S_p at q_k, denoted by $R(S_p, q_k)$, is*

$$\max_{i \in S_c} \textit{hops}(i, S_p, q_k)$$

where

$$\begin{aligned}
S_c &= \{i : i \in V.q_k \wedge \\
&\qquad i \textit{ is contaminated by } S_p\}, \\
\textit{hops}(i, S_p, q_k) &= \min_{j \in S_p} \textit{hops}(i, j, q_k), \\
\textit{hops}(i, j, q_k) &= \textit{the number of hops in a shortest} \\
&\qquad \textit{path between } i \textit{ and } j \textit{ in } G.q_k.
\end{aligned}$$

To prevent fault propagation and to increase the stability as well as the availability of networks in the presence of high-frequency faults, it is desirable that, at every state, the contamination range of each perturbed region remain bounded relative to the size of the perturbed region. Formally, this property is characterized as

Definition 5 ($\mathcal{F}$-containment) *A system is $\mathcal{F}$-containing if and only if*

> *For every perturbed region S_p at an arbitrary system state q_k $(k \geq 0)$, $R(S_p, q_k) = O(\mathcal{F}(|S_p|))$, where $\mathcal{F}$ is a function.*

Besides containing faults locally around where they occur, it is desirable that, once faults stop occurring (either indefinitely or for a long enough period), the network stabilizes quickly and, intuitively, within time depending on the perturbation size. Formally, this property is characterized as

Definition 6 ($\mathcal{F}$-stabilization) *A system is $\mathcal{F}$-stabilizing if and only if*

> *Starting at an arbitrary state q_k with an arbitrary history prefix $\mathcal{H}(q_k)$ and an arbitrary protocol execution $\mathcal{E}(q_k)$, the system computation is guaranteed to reach a legitimate state within $O(\mathcal{F}(\mathcal{P}(q_k, \mathcal{H}(q_k), \mathcal{E}(q_k))))$ time in the absence of faults, where $\mathcal{F}$ is a function.*

5 Protocol CPV

Our objective is to design a path-vector routing protocol that is both $\mathcal{F}$-containing and $\mathcal{F}$-stabilizing. In what follows, we first present the basic concepts underlying our protocol design, then we discuss the protocol CPV in detail.

5.1 Protocol concepts

One reason why faults propagate unboundedly in the example of Section 3 is as follows.

- When d rejoins the network, the route-withdrawal as the result of the fail-stop of d has already propagated to f.
- After d rejoins, e establishes its route to d and sends a route-announcement to f; the route-announcement, however, lags behind the route-withdrawal (which reflects an obsolete state of the network) in the sense that f has sent out the route-withdrawal to g before f receives the new route-announcement from e.
- As a result, the route-withdrawal keeps propagating from f to g, and then to h and i, even if the route-announcement tails it to reach g, h, and i.

Design pattern for continuous containment. To contain high-frequency faults, we, therefore, need a mechanism that enables information regarding each new network state to catch up with and stop the propagation of information regarding the preceding state which has become obsolete. To this end, we design protocol CPV where the diffusing computation involved in path-vector routing consists of three diffusing waves propagating at different speed:[5] *stabilization wave* at the lowest speed, *containment wave* at an intermediate speed, and *undo-containment wave* at the highest speed (see Figure 2). The three diffusing waves run in parallel and coordinate

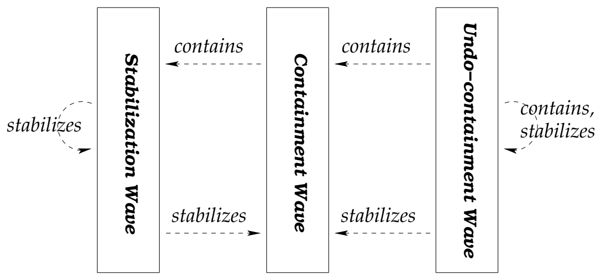


Figure 2: Parallel diffusing waves in CPV

to contain the propagation of obsolete information while stabilizing a network at the same time:

- Whenever a node j needs to change its state, it engages a containment wave cw_0 before engaging a new stabilization wave sw_1, so that cw_0 stops the previous stabilization wave sw_0 from propagating the existing state of j (which will become obsolete once j executes sw_1);
- In the presence of high-frequency faults, another fault f may occur before j executes sw_1, then there are two cases:
  - (i) j does not need to change its state any more after f occurs. Then cw_0 has to be stopped so that sw_0 can keep propagating to nodes whose state still needs to be corrected. To this end, j engages an undo-containment wave uw_0 which catches up with and stops cw_0.
  - (ii) j still needs to change its state after f occurs. Then j does nothing but lets cw_0 propagate (to stop sw_0).
- Each stabilization as well as undo-containment wave stabilizes itself, and each containment wave is stabilized (and deactivated) by the corresponding stabilization or undo-containment wave.

We elaborate on the design pattern as follows.

A. Containing a stabilization wave: When a node needs to engage a new stabilization wave to change its state, the existing state of the node becomes obsolete but may have propagated to other nodes via the previous stabilization wave. To avoid unbounded propagation of its

[5] As we will see later, the reason we need 3 diffusing waves is to guarantee that a "mistakenly" initiated containment wave will not propagate unboundedly. More concretely, since containment wave introduces new variables (i.e., *ghost* and *tp*, see Section 5.2) other than those used by stabilization wave, containment wave cannot guarantee the containment of itself because its associated variables can be corrupted. Therefore, we introduce undo-containment wave to contain any mistakenly initiated containment wave. Since undo-containment wave does not introduce any new variable, it self-contains and does not need another 4th diffusing wave.

existing state, the node engages a containment wave before generating the new stabilization wave, and the containment wave will stop the previous stabilization wave. When a containment wave propagates from a node i to its neighbor j, j further propagates the containment wave if j has propagated the obsolete information from i. For instance, in the example discussed in Section 3, when d rejoins, e will detect that it needs to change state (i.e., to establish a route to d); therefore, e initiates a containment wave toward f, then f decides whether to propagate the containment wave depending on whether f has propagated the route-withdrawal to g, and so on.

To contain stabilization waves in the presence of high-frequency faults, each containment wave from a node i carries the prediction of the state to which i will converge so that the neighbors of i are able to decide whether to hold a stabilization wave.[6] Moreover, to contain high-frequency faults, the containment and stabilization waves that are related to the same fault should be able to co-exist. To this end, containment waves do not modify variables of stabilization waves, and each containment wave is a one-way open (instead of a two-way closed) diffusing process that is deactivated later by an associated stabilization or undo-containment wave.

B. Containment-assisting stabilization wave: Stabilization waves are initiated or propagated by nodes that need to change routes after the occurrence of faults. To enable continuous fault containment, stabilization waves adapt the basic path-vector routing algorithm with the following mechanisms.

(i) When a node selects its route to a destination, it takes into account the predicted state of its neighbors.

(ii) When a containment wave catches up with the corresponding stabilization wave at a node i, both the containment and the stabilization wave stop at i (this is enabled by not letting i join any stabilization wave). For instance, in the example discussed in Section 3, after d rejoins, if the containment wave initiated at e reaches f before f propagates the route-withdrawal, f will not propagate the route-withdrawal anymore, which prevents the route-withdrawal from propagating further.

On the other hand, to guarantee convergence (especially in the presence of transient loops in path-vector routing [21]), a node that has already joined a containment wave is free to join a stabilization wave without sacrificing continuous fault containment.

(iii) Stabilization waves propagate slower than containment waves, so that containment waves can catch up with associated stabilization waves which are initiated earlier.

[6]This is discussed in detail in Section 5.2, more specifically, in the way notation $rank(i,k)$, protocol action SW, and predicate $S.i.j$ are defined. One fundamental reason why we need to use the *predicated information* is to guarantee the stabilization of the protocol CPV, especially the stabilization of the set of nodes that are involved in some containment wave. This is reflected in the way protocol actions SW and CW are defined. This detail does not show up at the level of protocol concepts, but it is a result of actually implementing the protocol concepts in a self-stabilizing distributed protocol.

C. Undoing a containment wave: In the presence of high-frequency faults, information that is obsolete at some point in time may become valid later when other faults occur. In this case, the containment wave marking that information as obsolete should be stopped and prevented from propagating further. This is enabled by letting the node that first detects this situation initiate an undo-containment wave which propagates along the same path as the corresponding containment wave. Since the undo-containment wave propagates faster than the containment wave does, the undo-containment wave is able to catch up with and stop the containment wave. Considering the example discussed in Section 3, for instance, when d fail-stops, a containment wave cw_0 will propagate from e toward f, and so on; when d rejoins later, e will initiate another containment wave cw_1 carrying the route-announcement that is "predicted" to take place later; suppose f has not withdrawn its route when it receives the predicted route-announcement from e, f will detect that it should stop cw_0; therefore, f initiates an undo-containment wave toward g to stop cw_0. (Note that f does not propagate cw_1, since f has not withdrawn its route.)

For the parallel-diffusing-wave based approach to work, each undo-containment wave must self-stabilize itself locally in the presence of faults (otherwise, we would need another type of diffusing wave to contain the undo-containment wave). This is achieved by ensuring that undo-containment waves use only those variables that are defined for stabilization and containment waves, but no other variable. Therefore, undo-containment waves are stateless, by which local stabilization is trivially guaranteed (since there does not exist any fault for a stateless component).

Summary. To sum up, the three-wave design is as follows.

- Containment waves contain the propagation of stabilization waves, and undo-containment waves contain the propagation of containment waves.
- Each contained wave (e.g., a stabilization wave) sets the boundary of its corresponding containing wave (e.g., a containment wave) such that the latter does not propagate beyond where the former has reached.
- Each containment wave is associated with two stabilization waves: the old stabilization wave which propagates obsolete information and which is contained, and the new stabilization wave which deactivates the containment wave after the latter has stopped the old stabilization wave.

5.2 The design of CPV

The protocol CPV is shown in Figure 3, where the protocol constants, variables, and actions for each node i are defined.

Constants. CPV uses five constants: d, $i.d_s$, $i.d_c$,

```
Protocol   $CPV.i$
Constant   $d$ : set of node-ids
           $i.d_s,\ i.d_c,\ i.d_u,\ I_{syn}$ : real
Var        $i.aspath,\ i.tp$ : list of AS-ids
           $i.j'.aspath,\ i.j'.tp\ (j' \in IM.i)$ : list of AS-ids
           $i.ghost,\ i.j'.ghost\ (j' \in IM.i)$ : boolean
           $i.t$ : real
           $k$ : node-id
Parameter  $j$ : node-id
Action
  $\langle SW \rangle :: S.i.j \vee R.i \vee R'.i \xrightarrow{i.d_s}$
            if $S.i.j \rightarrow i.aspath := aspath(i,j);$
                      $i.ghost := i.j.ghost$
              []
              $R.i \rightarrow$ if $i \in d \rightarrow i.aspath := [i.as]$
                                []
                                $i \notin d \rightarrow i.aspath := \emptyset$
                            fi;
                            $i.ghost := false$
            fi;
            $i.tp := \emptyset;$
            $i.t := CLK.i;$
            send $m(i.aspath, i.ghost, i.tp)$ to $EX.i$
  []
  $\langle CW \rangle :: (\neg i.ghost \wedge JC.i) \vee TP.i \xrightarrow{i.d_c}$
            $i.ghost := true;$
            do $S.i.k \rightarrow i.tp := aspath(i,k)$ od;
            do $S'.i.k \rightarrow i.tp := tp(i,k)$ od;
            $i.t := CLK.i;$
            send $m(i.ghost, i.tp)$ to $EX.i$
  []
  $\langle UW \rangle :: LC.i \vee CC.i \xrightarrow{i.d_u}$
            if $LC.i \rightarrow i.ghost := false$ fi;
            if $\neg R'.i \rightarrow i.tp := \emptyset$ fi;
            $i.t := CLK.i;$
            send $m(i.ghost, i.tp)$ to $EX.i$
  []
  $\langle SYN_1 \rangle :: (i.t + I_{syn} \leq CLK.i) \vee (i.t > CLK.i) \longrightarrow$
            $i.t := CLK.i;$
            send $m(i.aspath, i.ghost, i.tp)$ to $EX.i$
  []
  $\langle SYN_2 \rangle ::$ rcv $m$ from $j \longrightarrow$
            if $j \in IM.i \rightarrow$
              update $i.j.aspath$, $i.j.ghost$, and/or $i.j.tp$
            fi
```

Figure 3: CPV: Fault-Containing Path Vector Routing

$i.d_u$, and I_{syn}. d denotes the ID of the destination to which all other nodes in the network need to maintain a route; $i.d_s$, $i.d_c$, and $i.d_u$ are used to control the propagation speed of stabilization waves, containment waves, and undo-containment waves respectively; I_{syn} is used to control the frequency of information synchronization between neighboring nodes.

To contain high-frequency faults, $i.d_s$, $i.d_c$, and $i.d_u$ should satisfy the following equations:

$$i.d_s > i.\alpha \cdot (i.d_c + i.U) \tag{1}$$

$$i.d_c > i.\alpha \cdot (i.d_u + i.U) \tag{2}$$

$$i.d_u \geq 0 \tag{3}$$

where

$$i.U = \max_{(i,j)\in E} (i,j).U$$

(Details of the derivation can be found in the Appendix, more specifically, in the proof of Theorem 1.) Given today's high precision clocks (and clock synchronization in the Internet), $i.\alpha$ tends to be quite close to 1; given today's high speed networks, $i.U$ tends to be quite small, for instance, a few microseconds or milliseconds. Since $i.d_s$, $i.d_c$, and $i.d_u$ are usually more than several seconds, Formulae 1, 2, and 3 can be satisfied even if we conservatively set $i.\alpha$ and $i.U$ (to be large). Note that the above constants can be different for different nodes in the network, thus CPV is readily applicable irrespective of the degree of heterogeneity in the network.

Variables. As in BGP, node i maintains its AS-level path to d, denoted as $i.aspath$. To enable containment waves, i uses two additional variables: $i.ghost$ and $i.tp$. $i.ghost$ denotes whether or not i is involved in a containment wave, and $i.tp$ denotes the predicted route which i will adopt next. To coordinate with its neighbors, i also maintains a copy of the three variables for each of its import neighbors j', denoted as $i.j'.aspath, i.j'.ghost$, and $i.j'.tp$. (We discuss memory-efficient implementation of CPV in Section 8.)

For convenience, variable $i.t$ is used to record the time when i sends a message to its export neighbors the last time; a dummy variable k is also used.

Protocol actions. CPV consists of five actions, one for each of the diffusing waves involved in CPV and two for updating information between neighbors. We briefly explain each action by its category. For convenience, we define the following notations:

$aspath(i,k)$	:	$i.k.aspath$, if $i.as = k.as$
		$[i.as, i.k.aspath]$, if $i.as \neq k.as$;
$tp(i,k)$	:	$i.k.tp$, if $i.as = k.as$
		$[i.as, i.k.tp]$, if $i.as \neq k.as$;
$rank(i,k)$	:	$\max\{rank(i.k.aspath), rank(i.k.tp)\}$;
$N(i)$	:	the next-hop of i on its route to d;
$IM.i$	:	the set of import neighbors of i;
$EX.i$	:	the set of export neighbors of i;
$CLK.i$	:	the current clock value of i.

Stabilization wave. Stabilization waves are implemented by action SW. SW is executed when

- i needs to propagate a stabilization wave from an import neighbor j (i.e., $S.i.j = true$), i needs to reset $i.aspath$ (i.e., $R.i = true$), or i needs to reset $i.tp$ (i.e., $R'.i = true$); and
- the above condition has continuously held for the past $i.d_s$ time.

Corresponding to the intuitive formulation of stabilization waves in Section 5.1.B,

- $S.i.j \equiv$

$$\begin{array}{l}
i \notin d \wedge j \in IM.i \wedge i.j.aspath \neq \emptyset \wedge i.as \notin i.j.aspath \wedge \\
(i.N(i).aspath \neq \emptyset \wedge i.as \notin i.aspath \wedge \neg i.ghost \Rightarrow \\
\qquad\qquad \neg i.N(i).ghost) \wedge \\
((\neg i.j.ghost \wedge (\forall k : k \in IM.i \wedge k \neq j \Rightarrow \\
\qquad\qquad rank(i,k) < rank(i.j.aspath))) \vee \\
\ (i.j.ghost \wedge (\forall k : k \in IM.i \wedge k \neq j \Rightarrow \\
\qquad\qquad (i.k.aspath \neq \emptyset \Rightarrow i.k.ghost) \wedge \\
\qquad\qquad rank(i,k) < rank(i.j.aspath))) \\
) \wedge \\
(i.as \in i.aspath \vee (j = N(i) \wedge i.aspath \neq aspath(i,j)) \vee \\
\ (j \neq N(i) \wedge rank(i.N(i).aspath) < rank(i.j.aspath)))
\end{array}$$

- $R.i \equiv$

$$\begin{array}{l} (i \in d \wedge i.aspath \neq [d]) \vee \\ (i \notin d \wedge i.aspath \neq \emptyset \wedge \\ \quad (\forall k : k \in IM.i \Rightarrow (i.k.tp = \emptyset \vee i.as \in i.k.tp) \wedge \\ \qquad (i.k.aspath = \emptyset \vee i.as \in i.k.aspath))) \end{array}$$

- $R'.i \equiv$

$$\begin{array}{l} i.tp \neq \emptyset \wedge \\ (\forall k : k \in IM.i \Rightarrow (i.k.tp = \emptyset \vee i.as \in i.k.tp) \wedge \\ \qquad (i.k.aspath = \emptyset \vee i.as \in i.k.aspath)) \end{array}$$

To execute SW, i updates $i.aspath$ and $i.ghost$ appropriately. Since the execution of SW means fixing the route to a destination, i always reset $i.tp$ to empty, signifying that i will not change route any more unless faults occur again. After updating its state, i sends the updated state to its export neighbors.

Containment wave. Containment waves are implemented by action CW. CW is executed when

- i needs to join a containment wave (i.e, $\neg i.ghost \wedge JC.i$ holds), or i needs to update $i.tp$ (i.e., $TP.i = true$); and
- the above condition has continuously held for the past $i.d_c$ time.

Corresponding to the intuitive formulation of containment waves in Section 5.1.A,

- $JC.i \equiv$

$$\begin{array}{l} (i \in d \wedge i.aspath \neq [d]) \vee \\ (i \notin d \wedge ((\exists k : S.i.k) \vee R.i \vee \\ \qquad (i.N(i).ghost \wedge i.aspath = aspath(i, N(i))))) \end{array}$$

- $TP.i \equiv$

$$\begin{array}{l} (\exists k : S.i.k \wedge i.tp \neq aspath(i,k)) \vee \\ ((i.ghost \vee JC.i) \wedge \\ \quad (\exists k : S'.i.k \wedge i.tp \neq tp(i,k) \wedge i.aspath \neq tp(i,k))) \end{array}$$

where $S'.i.j \equiv$

$$\begin{array}{l} i \notin d \wedge j \in IM.i \wedge i.j.tp \neq \emptyset \wedge i.as \notin i.j.tp \wedge \\ (\forall k : k \in IM.i \wedge k \neq j \Rightarrow rank(i,k) < rank(i.j.tp)) \end{array}$$

To execute CW, i sets $i.ghost$ as $true$ to signify that it is involved in a containment wave and will change route soon. i also updates $i.tp$ as appropriate.

Undo-containment wave. Undo-containment waves are implemented by action UW. UW is executed when

- i is involved in a containment wave but does not need to change route any more (i.e., $LC.i = true$), or the part of the state of i related to containment waves has been corrupted (i.e., $CC.i = true$); and
- the above condition has continuously held for the past $i.d_u$ time.

Corresponding to the intuitive formulation of undo-containment waves in Section 5.1.C,

- $LC.i \equiv$

$$\begin{array}{l} i.ghost \wedge \\ ((\exists k : S'.i.k \wedge i.aspath = tp(i,k)) \vee \\ \quad (\neg JC.i \wedge \neg(\exists k : S'.i.k)) \vee (R'.i \wedge \neg R.i)) \end{array}$$

- $CC.i \equiv$

$$\begin{array}{l} (i.tp \neq \emptyset \wedge \neg(\exists k : S.i.k \vee S'.i.k)) \vee \\ (\neg i.ghost \wedge i.tp \neq \emptyset \wedge \neg R'.i) \vee i.as \in i.tp \end{array}$$

To execute UW, i resets $i.ghost$ to $false$, and if there is still a route to the destination (i.e., $R'.i = false$), i resets $i.tp$ to empty.

Note that, when d keeps fail-stopping and rejoining at high frequencies, it is critical to the fault containment that the reset of $i.tp$ be performed at the speed of stabilization waves instead of that of undo-containment waves; otherwise, the reset of $i.aspath$ could potentially propagate far away.

Information update. Actions SYN_1 and SYN_2 enable neighboring nodes to exchange information so that information consistency is guaranteed.

- Action SYN_1: if i has not updated its state with its export neighbors for more than I_{syn} time (i.e., $t.i + I_{syn} \leq CLK.i$) or if $i.t$ is corrupted to be greater than the current clock value, i sets $i.t$ to its current clock value, and sends its state (i.e., $i.aspath$, $i.ghost$, and $i.tp$) to its export neighbors.
- Action SYN_2: when i receives a state update from an import neighbor j, i updates its state record regarding j.

Note that actions SYN_1 and SYN_2 are needed only to tolerate potential state corruptions. Since state corruptions are rare events, SYN_1 and SYN_2 can be executed at low frequencies (e.g., once a day) or when the nodes are otherwise idle.

5.3 Example revisited

We reconsider the example discussed in Section 3 by examining how the network behaves if CPV is adopted. For simplicity of presentation, we assume that $i.\alpha = \alpha = 1$, $i.d_s = d_s$, $i.d_c = d_c$, and $i.d_u = d_u$ for every node i, link delay is a constant u, processing delay is negligible, and the propagation speed of undo-containment waves is twice that of containment waves, which in turn is twice that of stabilization waves (i.e., $d_s = 2d_c + u$ and $d_c = 2d_u + u$).

When d fail-stops, actions SW and CW are enabled at e since both of the predicates $R.e$ and $JC.e$ hold. Given that $d_c < d_s$, e initiates a containment wave cw_1 (by executing action CW) earlier than it initiates a stabilization wave sw_1. Now suppose sw_1 has reached f (i.e., action SW is enabled at f) and cw_1 has reached g (i.e., action CW is enabled at g) when d rejoins. After d rejoins, actions SW and CW are enabled at e again since both of the predicates $S.e.d$ and $JC.e$ hold. After $d_c + u$ time, the second containment wave cw_2 from e reaches f, carrying the predicted state of e (i.e., $e.tp$ as $[d.as]$); at the same time, cw_1 reaches h. When cw_2 reaches f, action SW becomes disabled at f (since $R.i$ becomes $false$), thus sw_1 stops at f; at the same time, action UW becomes enabled at f (since $LC.f$ holds), while CW remains disabled at e (which means that cw_2 stops at f). Since undo-containment waves propagate twice as fast as containment waves, the undo-containment wave uw_1 initiated from f will catch up with cw_1 at h, after which action CW becomes disabled at h and cw_1 as well as uw_1 stops at h.

In the above scenario, therefore, stabilization waves propagate to f the farthest, containment as well as undo-containment waves propagate to h the farthest, and node i is not affected. Moreover, packet forwarding is only slightly affected in the sense that only e changes its route in the presence of faults and that e recovers its route (via $e.tp$) as soon as d rejoins.

6 Analysis of CPV

Given a network topology $G'(V', E')$ and a routing policy function P', we let

$$\mathcal{L} \equiv (\forall i : i \in V' \Rightarrow \neg i.ghost \wedge i.tp = \emptyset \wedge LH.i)$$

where $LH.i \equiv$

$$\begin{aligned}
&(i \in d \Rightarrow i.aspath = \emptyset) \wedge \\
&(i \notin d \Rightarrow ((d \subseteq V' \Rightarrow i.aspath = aspath(i, hr(i, P'))) \wedge \\
&\qquad\qquad (d \not\subseteq V' \Rightarrow i.aspath = \emptyset)))
\end{aligned}$$

with $hr(i, P')$ being the highest-ranked neighbor of i, i.e.

$$(\forall k : k \in IM.i \wedge k \neq hr(i, P') \Rightarrow rank(i.k.aspath) < rank(i.hr(i, P').aspath))$$

Then, every state in $\mathcal{L}$ is a state where every up node in the network has found its best route to the destination. Thus every state in $\mathcal{L}$ is a legitimate state.

To analyze the properties of CPV, we first prove that CPV is self-stabilizing, then we prove that CPV contains high-frequency faults and locally stabilizes. (For clarity of presentation, we relegate the proofs of the theorems to the Appendix at the end of this paper.)

Lemma 1 (Self-stabilization) *Starting at an arbitrary state, every computation of a system where CPV is used is guaranteed to reach a state in $\mathcal{L}$.*

For cases where faults keep occurring at high frequencies, we have

Theorem 1 (Continuous containment) *In a system where CPV is used, the contamination range $R(S_p, q_k)$ of every perturbed region S_p at an arbitrary state q_k is $O(|S_p|)$. That is, a system where CPV is used is $\mathcal{F}$-containing, with $\mathcal{F}$ being a linear function.*

For cases where a node keeps changing its state (e.g., keeps fail-stopping and rejoining [6, 18, 26]), we have

Theorem 2 (Stability-adaptive control) *In a system where CPV is used, the distance to which a state $q_{i,k}$ of a node i propagates is $\Theta(t_{i,k})$, where $t_{i,k}$ is the sojourn time of state $q_{i,k}$.*

By Theorem 2, we see that, in CPV, the more unstable a node is, the shorter is the distance to which its state propagates (since the sojourn time of the state of a more unstable node is shorter). Therefore, network stability is improved without sacrificing network convergence in the sense that *a steady state reaches outwards, and a transient state stays.*

For cases where faults stop occurring (either indefinitely or for a long enough period) after a point in time, we have

Theorem 3 (Local stabilization) *Starting at an arbitrary state q_k with an arbitrary history prefix $\mathcal{H}(q_k)$ and an arbitrary protocol execution $\mathcal{E}(q_k)$, the system computation where CPV is used reaches a legitimate state within $O(\Gamma(\mathcal{P}(q_k, \mathcal{H}(q_k), \mathcal{E}(q_k))))$ time in the absence of faults, where Γ is a function reflecting the route ranking policy used in the system. That is, a system where CPV is used is Γ-stabilizing.*

In the common case where the SPF policy is used, Theorem 3 implies

Corollary 1 *A system where CPV and the SPF policy are used is Γ-stabilizing, and Γ is a linear function.*

By the analysis above, we see that CPV contains high-frequency faults and locally stabilizes; the degree of fault containment and the time taken to stabilize is a function of the perturbation size instead of the network size. We also see that CPV is "stability-adaptive" in the sense that the state of stable nodes propagate outwards, and the state of unstable nodes is locally contained.

In the next section, we corroborate our analysis via simulation with Internet-like networks.

7 Simulation results

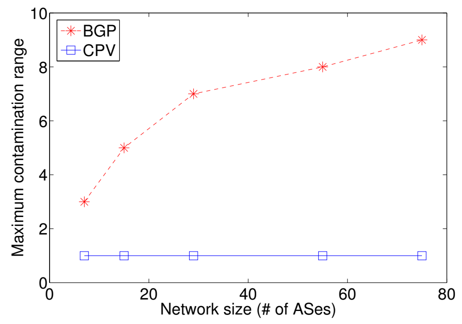
We have implemented CPV in SSFNet [1], a network simulator which supports a rich set of standard Internet protocols such as BGP (with route-flap-damping). In our simulation, CPV is executed without using any existing instability-suppression mechanisms. BGP is executed with instability-suppression timers and route-flap-damping (and the standard parameter configuration is used); sender-side-loop-detection is also used in BGP to reduce the probability of route-damping incurred delay in convergence [17]. For comparability, we set parameter d_s of CPV as 30 seconds, which is the default MinRouteAdvertisementInterval value used in BGP. Accordingly, we set d_c and d_u as 10 and 1 seconds respectively for CPV.

For fidelity of simulation, we use realistic Internet-type topologies [1] to evaluate CPV. And to study the impact of network size, we use networks of size ranging from 7 ASes to 75 ASes.[7]

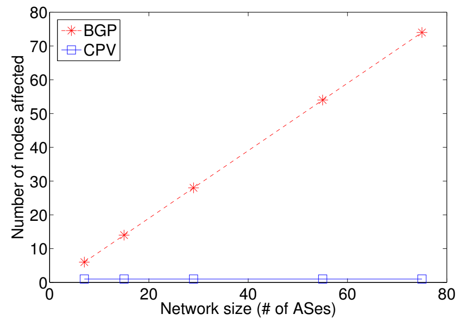
To simulate high-frequency faults, we let an arbitrary node j repeatedly fail-stop and then rejoin every 30 seconds. Since the impact of the faults at j is larger when j is the destination, we only present data for cases when j is the destination. The data presented in this section are averaged over 10 runs of simulation with different randomly selected j. We discuss the simulation results as follows.

Continuous containment. Figure 4 shows the maximum contamination range and the maximum number of nodes affected (i.e., perturbed or contaminated) by the faults in BGP and CPV. We see that the contamination range and the number of nodes affected by the faults in BGP increase as network size increases, and every node is affected by the faults in BGP; whereas in

[7]The benefits of guaranteed fault containment in CPV can be better demonstrated in networks of larger scale, but we are only able to run simulations of up to 75 ASes given the computing capabilities of our computer.



(a) Maximum contamination range



(b) The number of nodes affected

Figure 4: Impact of high-frequency faults

CPV, the contamination range remains 1 and the number of nodes affected by the faults remains small as network size increases, since CPV contains the impact of the high-frequency faults locally around where they occur (as proved in Theorem 1).

Local stabilization. Figure 5 shows the time taken for BGP and CPV to stabilize once (a) faults stop occurring when j is up, and (b) at lease one neighbor of j starts to announce the existence of j. We see that the time taken

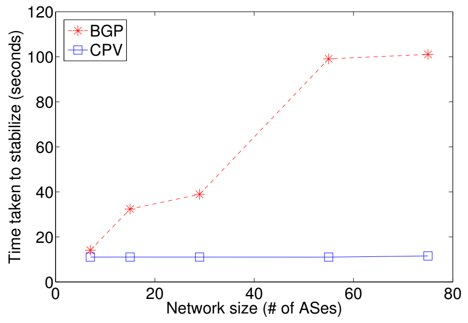


Figure 5: Time taken to stabilize

for BGP to stabilize increases as network size increases; whereas the time taken for CPV to stabilize remains small as network size increases, since the time taken for CPV to stabilize depends on the perturbation size instead of the network size (as proved in Theorem 3), and the perturbation size remains small in CPV as network size increases.

To study the property of stability-adaptiveness in CPV, we used a network of 75 ASes, and we let an arbitrary node oscillate among different states, with varying oscillation frequencies. The simulation results are as follows.

Stability-adaptive control. Figure 6 shows the relationship between the sojourn time of a state and the distance to which the state propagates (i.e., the maximum distance from the oscillating node to the nodes that adapt to the state). We observe that the states that last longer propagate farther and, accordingly, more nodes adapt behaviors; as the sojourn time of a state becomes short, the distance to which the state propagates decreases, and its impact is contained tightly (as proved in Theorem 2).

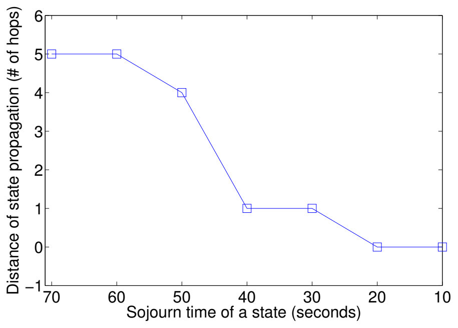


Figure 6: Stability-adaptiveness of CPV

8 Discussion

In this section, we discuss issues related to the application of CPV.

Sub-linear containment & stabilization. We mainly focused on the containment of high-frequency faults in this paper, and the contamination range as well as the convergence time (after faults stop occurring) is a linear function of the perturbation size in the common case where the SPF policy is used. If we apply, together with CPV, mechanisms that expedite the convergence of path-vector routing protocols (such as those proposed in [15], [5], [20], and [28]), then the contamination range of a perturbed-region is reduced to a linear function of the diameter of the region, and thus both the contamination range and the convergence time are reduced to a sub-linear function of the perturbation size.

Efficient implementation of CPV. Different from within BGP, a node i maintains two additional variables $i.ghost$ and $i.tp$ in CPV. $i.ghost$ is a boolean variable and can be encoded in a single bit, therefore $i.ghost$ does not incur much memory overhead. $i.tp$ denotes the next route that i will adopt when network state changes. Thus, $i.tp$ is transient and is meaningful only during the stabilization of CPV. Therefore, node i only needs to maintain $i.tp$ for destinations to which the route of i needs to change. Consequently, the use of $i.tp$ in CPV does not introduce much memory overhead either, since the impact of faults are locally contained in CPV and the majority of the network is stable most of the time.

In CPV, neighboring nodes exchange their state periodically, which is required for protocols to stabilize from state corruptions. To reduce the overhead of the periodic information synchronization, we can apply the technique proposed in [25] that guarantees the consistency of the routing-tables between neighboring nodes in a scalable manner. For instance, each round of synchronization between two neighbors only takes one message when there is no fault, and on average the overhead is only 1.3% of a full routing table exchange. Moreover, since state corruption does not happen frequently, information synchronization can be executed at low frequency (e.g., once a day) or when the nodes are otherwise idle. Therefore, the overhead incurred by information synchronization is almost negligible in practice.

Messaging complexity of CPV. In both CPV and BGP, the number of control messages required[8] depends on the number of nodes affected by faults and the associated edges.

In the presence of high-frequency faults in large scale networks, guaranteed fault containment in CPV significantly reduces the number of nodes that are affected by faults (e.g., by an order of magnitude as shown in Figure 4(b)). Therefore, CPV tends to significantly reduce the number of control messages, even though the introduction of containment and undo-containment waves in CPV can increase the number of control messages (by a factor up to 3) for the region affected by faults. For the high-frequency faults scenario discussed in Section 7, Figure 7 shows the number of control messages incurred in

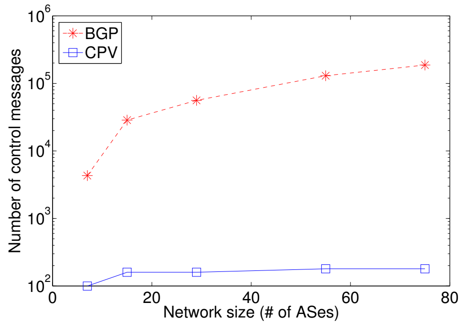


Figure 7: Messaging complexity

BGP and CPV respectively when each randomly chosen node j fail-stops and rejoins 20 times. We see that CPV significantly reduces the number of control messages, and the degree of reduction increases as the number of ASes increases (e.g., by three orders of magnitude for the network of 75 ASes).

In the case where faults happen only once or rarely, the containment waves may be wasted in CPV. To reduce the messaging complexity in this case, we can adapt CPV such that a node does not initiate containment waves[9] for faults that stopped T time ago, where T is an upper bound on the time taken for a network to stabilize after the faults (when the routing policies do not prevent the network from stabilizing). In practice, T can be conservatively chosen such that it bounds from above the stabilization time in most of the cases. For instance, we can set T as 30 minutes for the Internet where it usually takes no more than 15 minutes to stabilize from a fault in the absence of false BGP route-flap-damping [12, 17]. Even though this threshold-based approach does not eliminate wasteful containment waves in all cases, it is effective considering the fact that, in the case of low-frequency faults, the inter-fault interval at a node is usually much greater than tens of minutes [27]. Detailed study of this issue, however, is beyond the scope of this paper, and we relegate it as a part of the future work.

Incremental deployment of CPV. Containment waves in CPV introduce new information other than that used in BGP. To enable graceful migration of and interoperability with BGP, we define a new *optional transitive* path attribute [23], and use this attribute to encode the information newly introduced in CPV. According to BGP specification, a BGP speaker propagates a transitive attribute upon receipt, whether or not the BGP speaker implements CPV. Therefore, CPV can be incrementally deployed and inter-operate well with BGP. Moreover, even in the case of partial deployment, CPV can help contain faults that keep occurring to a region where CPV is used.

In the implementation of CPV, instability-suppression timers such as *MinRouteAdvertisementInterval* are not needed any more, since constant d_s in protocol action *SW* essentially performs similar roles as those timers. Route-flap-damping can be optimistically applied with CPV to detect mis-behaving nodes or links (to be discussed in more detail in Section 9).

9 Concluding remarks

To characterize system properties in the presence of high-frequency faults, we formulated the notions of perturbed node, contaminated node, perturbation size, contamination range, $\mathcal{F}$-containment, and $\mathcal{F}$-stabilization. These concepts are generically applicable to networking and distributed computing problems. In general, a self-stabilizing protocol that contains high-frequency faults also locally stabilizes.

We designed the path-vector routing protocol CPV that contains high-frequency faults, whether or not anticipated, and locally stabilizes. In CPV, the distance to which the state of a node propagates is proportional to the sojourn time of the state. CPV achieves these properties by layering a system computation into three diffusing waves (i.e., stabilization wave, containment wave, and undo-containment wave) which run in parallel and coordinate to contain the propagation of obsolete information

[8]Not considering the control messages exchanged between neighbors periodically.

[9]Accordingly, no undo-containment waves will be initiated.

while stabilizing a network at the same time. Built upon models for the Internet, CPV is readily applicable.

In this paper, we focused on how to contain high-frequency faults without detecting why and where the faults occur. This design enables CPV to work both when faults are benign [28], and when nodes are compromised in adversarial cases [18, 26]. In the latter cases, CPV can be applied together with security and fault diagnostic mechanisms that detect and suppress mis-behaving nodes or links; since CPV always contains faults locally around where they occur, the security and fault diagnostic mechanisms can be more optimistic and conservative (e.g., higher cutoff threshold and shorter suppression time in route flap damping [24]) to reduce the probability as well as the impact of false positive in fault detection (e.g., to avoid issues such as severely delayed convergence in BGP after route flap damping).

Acknowledgment

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Appendix: proofs of theorems

Lemma 1 (Self-stabilization) *Starting at an arbitrary state, every computation of a system where CPV is used is guaranteed to reach a state in $\mathcal{L}$.*

Proof: To prove this lemma, we first prove the stabilization property for the common case where the SPF policy is used; then we prove the case where general route ranking policy (which may not be the SPF policy) is used (by the help of existing results on the convergence properties of Simple Path Vector Protocols [19]).

`The SPF policy.`

First, we introduce the concept of "computation rounds" which will be used in the proof. A system computation $\mathcal{H}$ can be regarded as a sequence of *rounds*. A round is a minimal computation segment γ that starts at a state q_k ($k \geq 0$) and, in γ, (i) every up node that has an action a continuously enabled from some time t' to $(t' + d.a)$, where $t' \leq t_k$ and $(t' + d.a) \geq t_k$, executes at least one action, and (ii) if a message is sent to a node i, the action that receives the message must be executed at i. (We assume t_0 is the time when $\mathcal{H}$ starts.) For CPV, each round of a computation lasts at most $(d_s + U)$ time in length.

When the SPF policy is used, we prove that
[Claim 0] Starting at an arbitrary state, every computation of a system where CPV and the SPF policy are used is guaranteed to reach a state in $\mathcal{L}$ within $O(|V'|)$ time.

To prove Claim 0, we analyze the two different cases that can exist: when d is up (i.e., $d \subseteq V'$), and when d is down (i.e., $d \nsubseteq V'$).

[Claim 0.0] Starting at an arbitrary state when d is up (i.e., $d \subseteq V'$), every computation of a system where CPV and the SPF policy are used is guaranteed to reach a state in $\mathcal{L}$ within $O(D)$ time, where D is the number of hops in the longest shortest path from a node not in d to d in G'.

In this case, we prove the claim by the "convergence stair" approach used in [3] and some other works. That is, That is, we exhibit a finite sequence of state predicates $\mathcal{L}.0, \mathcal{L}.1, \ldots, \mathcal{L}.D$ such that

(i) Starting at an arbitrary state, the system is guaranteed to reach a state in $\mathcal{L}.0$.

(ii) For each l such that $0 \leq l \leq D$:
$\mathcal{L}.l$ is closed under system execution; that is, once $\mathcal{L}.l$ holds in an arbitrary system computation, it continues to hold subsequently.

(iii) For each l such that $0 \leq l < D$:
Upon starting at an arbitrary state in $\mathcal{L}.l$, the system is guaranteed to reach a state in $\mathcal{L}.(l+1)$ within a constant amount of time.

(iv) $\mathcal{L}.D \equiv \mathcal{L}$

For our protocol, $\mathcal{L}.0, \mathcal{L}.1, \ldots, \mathcal{L}.D$ are defined as follows,

- $\mathcal{L}.0 \equiv (\forall i : i \in d \Rightarrow i.aspath = \emptyset)$
- For $0 \leq l < D$:

$$\begin{aligned} \mathcal{L}.(l+1) \equiv\ & \mathcal{L}.l \wedge \\ & (\forall i : i \in V' \wedge hops(i,d,G') = l+1 \Rightarrow \\ & \qquad \neg i.ghost \wedge i.tp = \emptyset \wedge LH.i) \end{aligned}$$

where
$hops(i,d,G')$ = the number of hops in the shortest path from i to d in G'.

We prove the stabilization of $\mathcal{L}$ by proving the individual elements of the "convergence stair" method:

(i) Starting at an arbitrary state q_0, the system is guaranteed to reach a state in $\mathcal{L}.0$ within constant (i.e., d_s) time.

If $\mathcal{L}.0$ holds at state q_0, we are done.

If $\mathcal{L}.0$ does not hold at q_0, then $R.i$ must hold for some node $i \in d$. For each of such node i, therefore, action SW is enabled at q_0 and will be executed within d_s time.

Therefore, starting at an arbitrary state q_0, the system is guaranteed to reach a state in $\mathcal{L}.0$ within d_s time.

(ii) For each l such that $0 \leq l \leq D$: $\mathcal{L}.l$ is closed under system execution.

To prove this claim, we first prove that, starting at state q_0, all invalid routes that are of hop-length l' are removed within l' rounds of system computation. This comes from the fact that, for every such invalid route r, nodes in the ASes of r execute action SW which removes subpath r gradually. Therefore, this sub-claim is easily proved by induction on the number of hops from an AS in r to d. (For conciseness, we skip the detail here.)

Therefore, once the system reaches a state in $\mathcal{L}.l$, neither action SW nor action CW will be enabled for nodes whose minimum AS-level hop distance to d is less than or equal to l (i.e., $hops(i,d,G') \leq l$ for each of such node i), as a result of which no such node will change state any more. Thus, once the system reaches a state in $\mathcal{L}.l$, the system state remains in $\mathcal{L}.l$ subsequently.

(iii) For each l such that $0 \leq l < D$: Upon starting at an arbitrary state in $\mathcal{L}.l$, the system is guaranteed to reach a state in $\mathcal{L}.(l+1)$ within $(d_s + U)$ time.

To prove this, we only need to prove that for a node i where $hops(i,d,G') = l+1$ but $\neg i.ghost \wedge i.tp \wedge LH.i$ does not hold when the system is at state $\mathcal{L}.l$, $\neg i.ghost \wedge i.tp \wedge LH.i$ will become holding within $(d_s + U)$ time. This is true because, if $\neg i.ghost \wedge i.tp \wedge LH.i$ does not hold when the system is at state $\mathcal{L}.l$, $S.i.k = true$ for some import neighbor k of i where $hops(k,d,G') = l$; thus, action SW is enabled at i and will be executed within d_s time; given that it takes up to U time for the new state of k to reach i, i executes action SW within $d_s + U$ time after the system reaches a state in $\mathcal{L}.l$, by which $LH.i$ becomes $true$, $i.ghost$ is set to $false$, and $i.tp$ is set to $\emptyset$.

(iv) $\mathcal{L}.D \equiv \mathcal{L}$

This trivially holds since the maximum minimum-hop-distance from a node in V' to d is D.

Therefore, Starting at an arbitrary state when d is up (i.e., $d \subseteq V'$), every computation of a system where CPV and the SPF policy are used is guaranteed to reach a state in $\mathcal{L}$ within $O(D)$ time.

[Claim 0.1] Starting at an arbitrary state q_0 when d is done (i.e., $d \nsubseteq V'$), every computation of a system where CPV and the SPF policy are used is guaranteed to reach a state in $\mathcal{L}$ within $O(|V'|)$ time.

Claim 0.1 is easily proved by following the same approach as used in proving Claim 0.0 and in [13]. That is, by considering each route r that exists at state q_0, then nodes in the ASes of r execute action SW which removes subpath of r gradually. Therefore, by induction on the number of hops from an AS in r to d, we see that, starting at q_0, there will be no route in the system within $O(|V'|)$ rounds of computation, since the maximum hop-length of a loop-free route is $|V'|$, and every looping route is removed within constant time by executing action SW. Therefore, starting at state q_0, the system will reach a state where no node has a route to d (i.e., a state in $\mathcal{L}$) within $O(|V'|)$ time.

Having proved the lemma for the case where the SPF policy is used, we prove the lemma as follows for cases where non-SPF policies are used.

`Non-SPF policies.`

To prove the convergence of CPV from an arbitrary state to a state in $\mathcal{L}$ when route ranking policies other than the SPF policy is used, we first prove that

[Claim 1] starting at an arbitrary state q_k, the system reaches a state, within constant time, where the state related to containment waves and stabilization waves is consistent with each other, that is, a state in $\mathcal{L}_0$, with $\mathcal{L}_0$

defined as $(\forall i : CNST0.i \wedge CNST1.i)$, where $CNST0.i$ is defined as

$$
\begin{array}{l}
i.tp \neq \emptyset \Rightarrow \\
\quad (i.ghost \wedge \\
\quad\quad (\neg TP.i \Rightarrow \\
\quad\quad\quad (\exists k : (S.i.k \wedge i.tp = aspath(i,k)) \vee (S'.i.k : i.tp = tp(i,k)))))
\end{array}
$$

and $CNST1.i$ is defined as

$$i.ghost \Rightarrow JC.i \vee (\exists k : S'.i.k)$$

To prove Claim 1, we first prove as follows that, starting at an arbitrary state q_k, the system reaches a state where $(\forall i : CNST0.i)$ holds:

- Starting at an arbitrary state q_k, the system reaches a state where $(\forall i : i.tp \Rightarrow i.ghost)$ holds within constant time.

  For a node i where $i.tp \neq \emptyset \wedge \neg i.ghost$ holds, action UW is enabled if $R'.i = false$ (since $CC.i$ holds) and action SW is enabled if $R'.i = true$. Therefore, i executes UW within d_u time or executes SW within d_s time, as a result of which $i.tp$ is set to $\emptyset$ and thus $i.tp \Rightarrow i.ghost$ becomes hold for i.
- Starting at an arbitrary state where $(\forall i : i.tp \Rightarrow i.ghost)$ holds, the system reaches a state where $(\forall i : CNST0.i)$ holds within constant time.

  For a node i where $i.tp \neq \emptyset \wedge \neg TP.i \wedge \neg(\exists k : (S.i.k \wedge i.tp = aspath(i,k)) \vee (S'.i.k \wedge i.tp = tp(i,k)))$ holds, action UW is enabled at i since $CC.i = true$. Therefore, i executes UW within d_u time, as a result of which $i.tp = \emptyset$ and thus $CNST0.i$ holds.

Then we prove that, starting at an arbitrary state where $(\forall i : CNST0.i)$ holds, the system reaches a state in $(\forall i : CNST0.i \wedge CNST1.i)$ within constant time:

For a node i where $CNST0.i \wedge i.ghost \wedge \neg JC.i \wedge \neg(\exists k : S'.i.k)$ (i.e., $CNST0.i = true$ but $CNST1.i = false$), action UW is enabled since $LC.i = true$. Therefore, i executes UW within d_u time, as a result of which $CNST0.i \wedge CNST1.i$ holds.

Therefore, Claim 1 holds; then we prove that

[Claim 2] starting at an arbitrary state $q'_k in \mathcal{L}_0$, the system reaches a state in $\mathcal{L}$ within $O(\Gamma(|V'|))$ time, where $\Gamma(|V'|)$ depends on the route ranking policy used in a network and is the diameter of the timed dispute digraph of the system [19].

To prove Claim 2, we first prove that, starting at an arbitrary state in $\mathcal{L}_0$ where $\mathcal{L}$ does not hold, action SW will be executed at a node within constant time. In a state $\mathcal{L}_0$ where $\mathcal{L}$ does not hold, there can be two cases:

- There is no node i such that $i.ghost = true$:

  In this case, there must be a node i such that $(\exists k : S.i.k) \vee R.i$ holds (otherwise, the system would be in state $\mathcal{L}$). Therefore, node i will execute action SW within d_s time, unless another neighboring node executes SW which disables $(\exists k : S.i.k) \vee R.i$. Either way, there is at least a node executing SW within d_s time.
- There is at least one node i such that $i.ghost = true$:

  At a state in $\mathcal{L}_0$ where there is some node i such that $i.ghost = true$, there must be a node j such that $(\exists k : S.j.k) \vee R.j$ holds (otherwise, there would be no node in any containment wave). Therefore, node j will execute action SW within d_s time, unless another neighboring node executes SW which disables $(\exists k : S.j.k) \vee R.j$. Either way, there is at least a node executing SW within d_s time.

Thus, starting at a state q'_k in $\mathcal{L}_0$, there will be an action SW executed within every constant time until the system reaches a state in $\mathcal{L}$. By theorems in [19], if an action SW is executed at time t, which leads to a new system state $q_{k''}$, then there must exist a path from q'_k to $q_{k''}$ in the Timed Dispute Graph $TDD(G')$ of the system. Therefore, starting at a state $q'_k in \mathcal{L}_0$, the system reaches a state in $\mathcal{L}$ within $O(\Gamma(|V'|))$ time, where $\Gamma(|V'|)$ is the diameter of the timed dispute digraph of the system.

Therefore, Claim 2 holds; to finish the proof, we only need to prove that

[Claim 3] once a system reaches a state in $\mathcal{L}$, the system remains in $\mathcal{L}$ in the absence of faults.

Claim 3 trivially holds, since none of actions SW, CW, and UW is enabled at any node at a state in $\mathcal{L}$.

Therefore, Lemma 1 holds for cases where policies other than the SPF policy are used. That is, starting at an arbitrary state, every computation of a system where CPV is used is guaranteed to reach a state in $\mathcal{L}$ within $O(\Gamma(|V'|))$ time, where $\Gamma(|V'|)$ is the diameter of the Timed Dispute Digraph $TDD(G')$ of the system (which depends on the route ranking policy used).

□

Theorem 1 (Continuous containment) *In a system where CPV is used, the contamination range $R(S_p, q_k)$ of every perturbed region S_p at an arbitrary state q_k is $O(|S_p|)$. That is, a system where CPV is used is $\mathcal{F}$-containing, with $\mathcal{F}$ being a linear function.*

Proof: To prove this theorem, we first consider the case where there is only one perturbed region in the system, then we consider the case where multiple perturbed regions co-exist.

[Single perturbed region] In CPV, for every node i, only action SW and CW could propagate the effect of faults, since only stabilization and containment waves propagate the state changes at a node away from the node (whereas undo-containment waves only undo mistakenly-propagated containment waves, and undo-containments waves do not maintain any variable at all). Therefore, to prove this theorem when there is a single perturbed region, we need to prove that, for any perturbed region S_p, its contamination range is always $O(\Gamma(|S_p|))$ irrespective the execution and the system history prefix. There are two cases that can propagate the impact of the perturbed region during protocol execution that tries to stabilize the network from the perturbed state by the faults; we prove that in neither case will the propagation be unbounded.

- Case one: there is a node i that has a neighbor in S_p, and i has just initiated a containment wave (e.g., due to fast state oscillation) when i turns out not to initiate any stabilization wave.

  For every such node i, action UW is enabled since $LC.i = true$. Therefore, an undo-containment wave will be initiated that propagates along the same direction/paths as the containment wave, as shown in Figure 8.

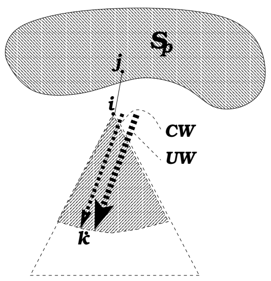

Figure 8: Containment wave in the absence of accompanying stabilization wave

  Because the propagation speeds of different diffusing waves are controlled by introducing delays in action execution when a node schedules its enabled actions, and there is difference between clock rates at different nodes, $j.d_c > j.\alpha \cdot (j.d_u + j.U)$ should hold for every node j in order for the undo-containment wave to catch up with the containment wave. Under this condition, the farthest distance dt (in terms of the number of hops) the containment wave could propagate along any path satisfies the following formula: $\sum_{j=1}^{dt}(j.U + j.d_u) + C0 = \sum_{j=1}^{dt}(j.L + j.d_c)$, where $C0$ is the delay in i initiating the undo-containment wave, and j is (the index of) a node in the path where CW has reached, $j.U = \max_{(j,k)\in E}(j,k).U$, and $j.L = \max_{(j,k)\in E}(j,k).L$. Thus, $dt = \theta(C0)$ and is a constant.

  Therefore, the theorem holds in this case.

- Case two: there is a node i that has a neighbor in S_p, and i has initiated a containment wave $CW0$ because i needs to initiate a stabilization wave $SW0$ by protocol CPV (see Figure 9).

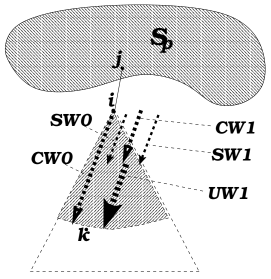

Figure 9: Coexisting containment and stabilize waves

In the presence of high-frequency faults, there are two subcases:

  - If i needs to change state after another fault f occurs, then every node whose route goes through i will need to change state anyway, no matter whether there is another stabilization wave $SW0'$ other than $SW0$ to be propagated by i.

    In this case, all the nodes that $CW0$ could have reached are perturbed nodes and need to change state. Therefore, the existence of $CW0$ and $SW0$ does not introduce any contaminated nodes, and the theorem trivially holds.

  - If i does not need to change state in the presence of faults that may occur to nodes in S_p later, then action CW and SW will be enabled at i later when nodes in the perturbed region change their state. The latest for this to occur is at most $O(|S_p|)$ time after $SW0$ is initiated by i, since a containment wave must be initiated in S_p and it takes at most $O(|S_p|)$ time for the containment wave to reach i (note that the hop-length of the longest path segment in S_p is $|S_p|$).

    When action CW is enabled i, a containment wave $CW1$ will be initiated/propagated which propagates along the same paths taken by $CW0$ and $SW0$. Given that containment waves propagate faster than stabilization waves do, $CW1$ will catch up with and stop $SW0$, at which point an undo-containment wave $UW1$ will be initiated to catch up with $CW0$. When node i executes action SW, i initiates another stabilization wave $SW1$ that stabilizes the state of nodes which have been affected by $SW0$ and $CW1$. No matter when $SW1$ will be initiated, the maximum distance $CW0$ and $SW0$ could propagate is bounded by the point where $UW1$ catches up with $CW0$.

    In order for $CW1$ to be able to catch up with and stop $SW0$ in the above process, $k.d_s > k.\alpha \cdot (k.d_c + k.U)$ should hold for every node k to which $SW0$ has reached, given that the propagation speeds of different diffusing waves are controlled by introducing delays in action execution when a node schedules its enabled actions, and that there is difference between clock rates at different nodes. Therefore, the farthest distance dt that $CW0$ and $SW0$ could propagate before being stopped satisfies the following formula: $O(|S_p|) + (\sum_{k=1}^{dt}(k.U + k.d_c)) \times (\max_k \frac{k.U+k.d_c}{k.L+k.d_s}) + (\sum_{k=1}^{dt}(k.U + k.d_u)) \times (1 - \max_k \frac{k.U+k.d_c}{k.L+k.d_s}) = \sum_{k=1}^{dt}(k.L + k.d_c)$, where k is (the index of) a node along the path where $CW0$ has reached. Thus $dt = O(|S_p|)$.

  Therefore, the theorem holds in this case.

`[Multiple perturbed regions]` When there are multiple perturbed regions, we need to prove that there is no interference between different perturbed regions. Toward this end, we first define the contamination region of a perturbed region S as the set of nodes that are contaminated by S.

Then, if the the contamination regions of two perturbed regions are disjoint, the claim holds trivially since they are unable to affect each other at all. Therefore, we only consider the case where the contamination regions of two or more perturbed regions are adjoining one another.

Without loss of generality, let us consider two perturbed regions $S0$ and $S1$, whose contamination regions are $CS0$ and $CS1$ respectively, and $CS0$ and $CS1$ are adjoining each other. To prove that $S0$ and $S1$ do not interfere each other, we only need to prove that, without loss of generality, the containment patterns as shown in Figures 8 and 9 for $S0$ are not affected by any node $k0$ that is contaminated by $S0$ but has a neighbor $k1$ that is contaminated by $S1$. This is the case because

- On one hand, if $k0$ does not change state because of the state changes at $k1$, then the existence of $S1$ does not affect the protocol execution within the contamination region of $S0$, and thus $S1$ does not affect the containment activities (by the execution of action CW and UW in the contamination region of $S0$) for $S0$;
- On the other hand, if $k0$ changes state because of the state changes at $k1$ and this interrupts the execution of containing action CW or UW in the contamination region of $S0$, this must be due to the fact that the corresponding protocol action CW or UW becomes disabled at $k0$ due to the state changes at $k1$. Therefore, $k0$ is regarded as contaminated by $S1$, and $k0$ will be involved in the containing action CW or UW in the contamination region of $S1$. Thus, the state changes at $k0$ will be contained too even in this case.

Therefore, the theorem holds when there are multiple perturbed regions at a state.

□

Theorem 2 (Stability-adaptive control) *In a system where CPV is used, the distance to which a state $q_{i,k}$ of a node i propagates is $\Theta(t_{i,k})$, where $t_{i,k}$ is the sojourn time of state $q_{i,k}$.*

Proof: To prove this theorem, let us consider an arbitrary pair of consecutive states $q_{i,k}$ and $q_{i,k+1}$ for a node i where the sojourn time for $q_{i,k}$ is $t_{i,k}$.

When i changes to state $q_{i,k}$, i initiates a stabilization wave $SW_{i,k}$ (in addition to a containment wave $CW_{i,k-1}$ that is to contain the propagation of the immediately previous state to $q_{i,k}$ at node i). When i changes from state $q_{i,k}$ to $q_{i,k+1}$ after $t_{i,k}$ time, $SW_{i,k}$ would have propagated up to $\frac{t_{i,k}}{d_s+L}$ hops. When i changes to state $q_{i,k+1}$, i initiates a containment wave $CW_{i,k}$ to catch up with $SW_{i,k}$. The farthest distance dt from a node where $CW_{i,k}$ catches up with $SW_{i,k}$ to i satisfies formula: $\sum_{j=1}^{dt}(j.d_s + j.L) = t_{i,k} + \sum_{j=1}^{dt}(j.d_c + j.U))$, where j is (the index of) a node that $CW_{i,k}$ has reached. Therefore, $dt = \Theta(t_{i,k})$. Similarly, the shortest distance dt' from a node where $CW_{i,k}$ catches up with $SW_{i,k}$ to i satisfies formula: $\sum_{j=1}^{dt'}(j.d_s + j.U) = t_{i,k} + \sum_{j=1}^{dt'}(j.d_c + j.L))$. Therefore, $dt = \Theta(t_{i,k})$.

Thus, the theorem holds.

□

Theorem 3 (Local stabilization) *Starting at an arbitrary state q_k with an arbitrary history prefix $\mathcal{H}(q_k)$ and an arbitrary protocol execution $\mathcal{E}(q_k)$, the system computation where CPV is used reaches a legitimate state within $O(\Gamma(\mathcal{P}(q_k, \mathcal{H}(q_k), \mathcal{E}(q_k))))$ time in the absence of faults, where Γ is a function reflecting the route ranking policy used in the system. That is, a system where CPV is used is Γ-stabilizing.*

Proof: Similar to the proof for Theorem 1, we prove this theorem by considering the two different cases depending on whether or not there is only one perturbed region in the system.

`[Single perturbed region]` When there is only one perturbed region S_p in the network, the time taken for nodes in S_p to stabilize is $O(\Gamma(|S_p|))$ according to the proof for Lemma 1 and Theorem 1.

By Theorem 1, the contamination range of S_p is $O(\Gamma(|S_p|))$. Since the time taken for the nodes in the contamination region of S_p to stabilize if proportional to the contamination range of S_p, the time taken for nodes contaminated by S_p to stabilize is $O(\Gamma(|S_p|))$.

When the time taken for the system to stabilize is $O(\Gamma(|S_p|)) + O(\Gamma(|S_p|)) = O(\Gamma(|S_p|))$. When S_p is the only perturbed region in the network, the perturbed size $\mathcal{P}(q_k, \mathcal{H}(q_k), \mathcal{E}(q_k)) = |S_p|$. Therefore, the system stabilizes within $O(\Gamma(\mathcal{P}(q_k, \mathcal{H}(q_k), \mathcal{E}(q_k))))$ time, and the claim holds in this case.

`[Multiple perturbed regions]` When there are multiple perturbed regions $S.0$, $S.1$, ..., and $S.m$, the time taken for the network to stabilize depends on whether the contamination regions of these perturbed regions are disjoint or not.

When the contamination regions of $S.0$, $S.1$, ..., and $S.m$ are disjoint, the stabilization of each contamination region is independent of that of the other contamination regions. Therefore, the time taken for the network to stabilize is $O(\max_{i=0..m}\Gamma(|S.i|))$. Since $\mathcal{P}(q_k, \mathcal{H}(q_k), \mathcal{E}(q_k)) = \sum_{i=0}^{m}|S.i|$ and $\Gamma(\sum_{i=0}^{m}|S.i|) \geq \max_{i=0..m}\Gamma(|S.i|)$ (by the definition of Timed Dispute Graphs [19]), the network stabilizes within $O(\Gamma(\mathcal{P}(q_k, \mathcal{H}(q_k), \mathcal{E}(q_k))))$ time.

When the contamination regions of two perturbed regions $S.k$ and $S.(k+1)$ are adjoining, the stabilization of one perturbed-region may depend on that of the other, because the stabilization in path-vector routing is essentially a diffusing computation from the destination. That is, one perturbed region as well as its contamination region can stabilize only after the other perturbed region as well as its contamination region has stabilized. Therefore, in the worst case where the set of perturbed regions as well as their contamination regions stabilize sequentially, the time taken for the system to stabilize is the summation of the stabilization time for each perturbed regions, i.e., $O(\sum_{i=0}^{m}\Gamma(|S.i|))$. Given that $\mathcal{P}(q_k, \mathcal{H}(q_k), \mathcal{E}(q_k)) = \sum_{i=0}^{m}|S.i|$ and $\Gamma(\sum_{i=0}^{m}|S.i|) \geq \sum_{i=0}^{m}\Gamma(|S.i|)$ (by the definition of Timed Dispute Graphs and the fact that Γ represents the length of the longest simple path in the Timed Dispute Graph [19]), the network stabilizes within $O(\Gamma(\mathcal{P}(q_k, \mathcal{H}(q_k), \mathcal{E}(q_k))))$ time.

Therefore, the theorem holds.

$\square$

Corollary 1 *A system where CPV and the SPF policy are used is* Γ*-stabilizing, and* Γ *is a linear function.*

Proof: From the proof for Theorem 3, we see that the time taken for a perturbed region S_p as well as its contamination region to stabilize is determined by the time taken for S_p to stabilize, which is $O(\Gamma(|S_p|)$ in general and Γ is a function depending on the route ranking policy used in the system. By the proof for Lemma 1, we see that, when the SPF policy is used, the time taken for a perturbed region S_p to stabilize is $O(|S_p|)$. Therefore, this corollary trivially holds.

$\square$